NON-RIGHT TRIANGLES 5.3.1 – 5.3.3

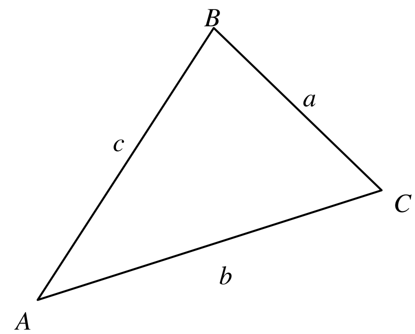
Students have several tools for finding parts of right triangles, including the Pythagorean Theorem, the tangent ratio, the sine ratio, and the cosine ratio. These relationships only work, however, with *right* triangles. What if the triangle is not a right triangle? Can we still calculate lengths and angles with trigonometry from certain pieces of information? Yes, by using two laws, the Law of Sines and the Law of Cosines that state:

Law of Sines

$$\frac{\sin(m\angle A)}{a} = \frac{\sin(m\angle B)}{b}$$

$$\frac{\sin(m\angle B)}{b} = \frac{\sin(m\angle C)}{c}$$

$$\frac{\sin(m\angle A)}{a} = \frac{\sin(m\angle C)}{c}$$



Law of Cosines

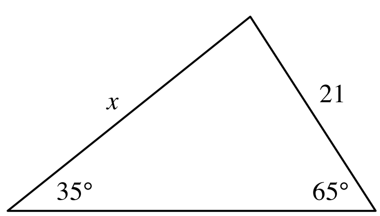
$$c^2 = a^2 + b^2 - 2ab\cos C \qquad b^2 = a^2 + c^2 - 2ac\cos B \qquad a^2 = b^2 + c^2 - 2bc\cos A$$

See the Math Notes boxes in Lessons 5.3.2 and 5.3.3.

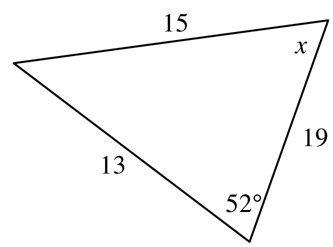
Example 1

Using the Law of Sines, calculate the value of x.

a.



b.



We will set up ratios that are equal according to the Law of Sines. The ratio compares the sine of the measure of an angle to the length of the side opposite that angle. In part (a), 21 is the length of the side opposite the 35° angle, while x is the length of the side opposite the 65° angle. The proportion is shown at right. To solve the proportion, we cross multiply, and solve for x. We can use the Law of Sines to find the measure of an angle as well. In part (b), we again write a proportion using the Law of Sines.

$$\frac{\sin 35°}{21} = \frac{\sin 65°}{x}$$

$$x \sin 35° = 21 \sin 65°$$

$$x = \frac{21 \sin 65°}{\sin 35°}$$

$$x \approx 33.18$$

$$\frac{\sin x}{13} = \frac{\sin 52°}{15}$$

$$15 \sin x = 13 \sin 52°$$

$$\sin x = \frac{13 \sin 52°}{15}$$

$$\sin^{-1} x = 13 \sin 52° \div 15$$

$$x \approx 43.07°$$

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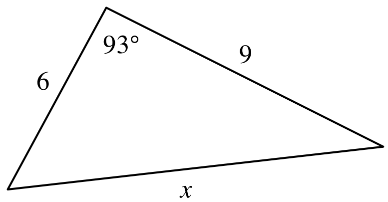
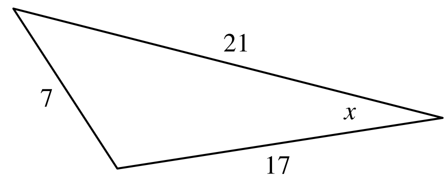
Example 2

Use the Law of Cosines to solve for x in the triangles below.

a.



b.



The Law of Cosines does not use ratios, as the Law of Sines does. Rather, it uses a formula somewhat similar to the Pythagorean Theorem. For part (a) the formula gives us the equation and solution shown below.

$$
\begin{aligned}
x^2 &= 6^2 + 9^2 - 2(6)(9)\cos 93^\circ \\
x^2 &\approx 36 + 81 - 108(-0.052) \\
x^2 &\approx 117 + 5.612 \\
x^2 &\approx 122.612 \\
x &\approx 11.07
\end{aligned}
$$

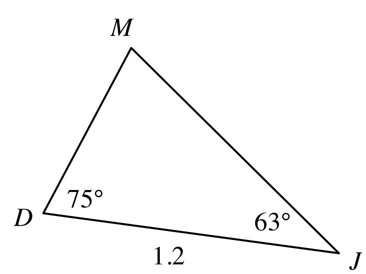
Just as with the Law of Sines, we can use the Law of Cosines to find the measures of angles as well as side lengths. In part (b) we will use the Law of Cosines to find the measure of angle x. From the law we can write the equation and solution shown below.

$$
\begin{aligned}
7^2 &= 17^2 + 21^2 - 2(17)(21)\cos x \\
49 &= 289 + 441 - 714\cos x \\
49 &= 730 - 714\cos x \\
-681 &= -714\cos x \\
\tfrac{-681}{-714} &= \cos x \\
x &\approx 17.49^\circ \text{ (using } \cos^{-1} x\text{)}
\end{aligned}
$$

Example 3

Marisa's, June's, and Daniel's houses form a triangle. The distance between June's and Daniel's houses is 1.2 km. Standing at June's house, the angle formed by looking out to Daniel's house and then to Marisa's house is 63°. Standing at Daniel's house, the angle formed by looking out to June's house and then to Marisa's house is 75°. What is the distance between all of the houses?

The trigonometry ratios and laws are very powerful tools in real world situations. As with any application, the first step is to draw a picture of the situation. We know the three homes form a triangle, so we start with that. We already know one distance: the distance from June's house to Daniel's house. We write 1.2 as the length of the side from D to J. We also know that $m\angle J = 63^\circ$ and $m\angle D = 75^\circ$, and can figure out that $m\angle M = 42^\circ$. We are trying to find the lengths of $\overline{DM}$ and $\overline{MJ}$. To do this, we will use the Law of Sines.



MJ:
$$
\begin{aligned}
\tfrac{\sin 75^\circ}{MJ} &= \tfrac{\sin 42^\circ}{1.2} \\
1.2\sin 75^\circ &= (MJ)\sin 42^\circ \\
\tfrac{1.2\sin 75^\circ}{\sin 42^\circ} &= MJ \\
MJ &\approx 1.73 \text{ km}
\end{aligned}
$$

DM:
$$
\begin{aligned}
\tfrac{\sin 63^\circ}{DM} &= \tfrac{\sin 42^\circ}{1.2} \\
1.2\sin 63^\circ &= (DM)\sin 42^\circ \\
\tfrac{1.2\sin 63^\circ}{\sin 42^\circ} &= DM \\
DM &\approx 1.60 \text{ km}
\end{aligned}
$$

Therefore the distances between the homes are: From Marisa's to Daniel's: 1.6 km, from Marisa's to June's: 1.73 km, and from Daniel's to June's: 1.2 km.

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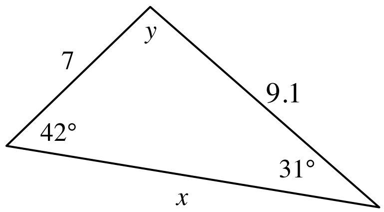
Problems

Use the tools you have for triangles to solve for x, y, or θ. Round all answers to the nearest hundredth.

1.



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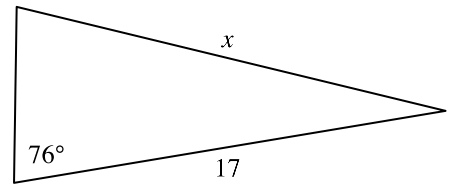

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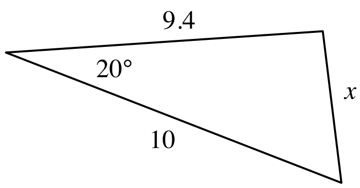

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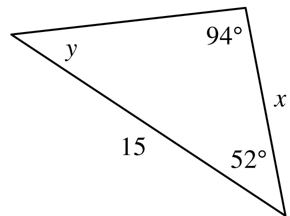

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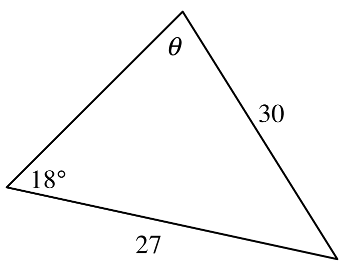

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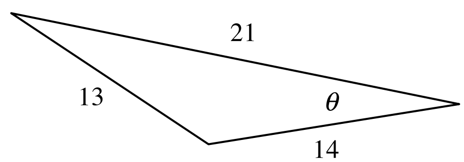

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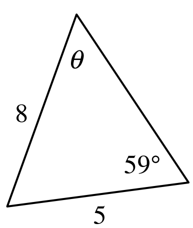

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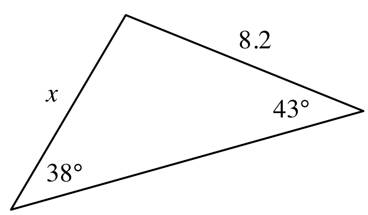

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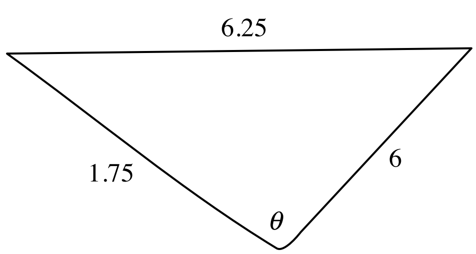

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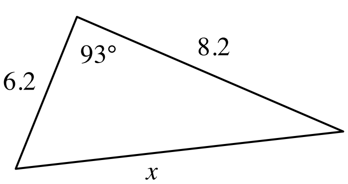

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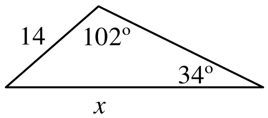
Use the Law of Sines or the Law of Cosines to find the required part of the triangle.

11.


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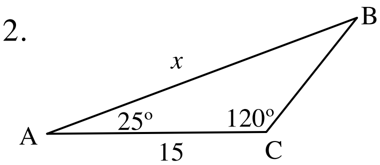

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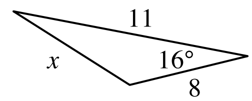

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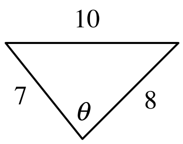

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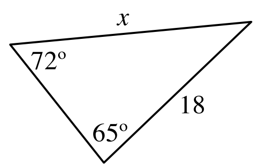

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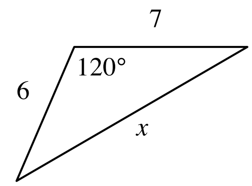

Draw and label a triangle similar to the one in the examples. Use the given information to find the required part(s).

17. $m\angle A = 40^\circ$, $m\angle B = 88^\circ$, $a = 15$.
Find b.

18. $m\angle B = 75^\circ$, $a = 13$, $c = 14$.
Find b.

19. $m\angle B = 50^\circ$, $m\angle C = 60^\circ$, $b = 9$.
Find a.

20. $m\angle A = 62^\circ$, $m\angle C = 28^\circ$, $c = 24$.
Find a.

21. $m\angle A = 51^\circ$, $c = 8$, $b = 12$.
Find a.

22. $m\angle B = 34^\circ$, $a = 4$, $b = 3$.
Find c.

23. $a = 9$, $b = 12$, $c = 15$.
Find $m\angle B$.

24. $m\angle B = 96^\circ$, $m\angle A = 32^\circ$, $a = 6$.
Find c.

25. $m\angle C = 18^\circ$, $m\angle B = 54^\circ$, $b = 18$.
Find c.

26. $a = 15$, $b = 12$, $c = 14$.
Find $m\angle C$.

27. $m\angle C = 76^\circ$, $a = 39$. B = 19.
Find c.

28. $m\angle A = 30^\circ$, $m\angle C = 60^\circ$, $a = 8$.
Find b.

29. $a = 34$, $b = 38$, $c = 31$.
Find $m\angle B$.

30. $a = 8$, $b = 16$, $c = 7$.
Find $m\angle C$.

31. $m\angle C = 84^\circ$, $m\angle B = 23^\circ$, $c = 11$.
Find b.

32. $m\angle A = 36^\circ$, $m\angle B = 68^\circ$, $b = 8$.
Find a and c.

33. $m\angle B = 40^\circ$, $b = 4$, and $c = 6$.
Find a, $m\angle A$, and $m\angle C$.

34. $a = 2$, $b = 3$, $c = 4$.
Find $m\angle A$, $m\angle B$, and $m\angle C$.

35. Marco wants to cut a sheet of plywood to fit over the top of his triangular sandbox. One angle measures 38°, and it is between sides with lengths 14 feet and 18 feet. What is the length of the third side?

36. From the planet Xentar, Dweeble can see the stars Quazam and Plibit. The angle between these two sites is 22°. Dweeble knows that Quazam and Plibit are 93,000,000 miles apart. He also knows that when standing on Plibit, the angle made from Quazam to Xentar is 39°. How far is Xentar from Quazam?

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Answers

1. $x \approx 13.00, y = 107^\circ$	2. $x \approx 16.60$
3. $x \approx 3.42$	4. $x \approx 8.41, y = 34^\circ$
5. $\theta \approx 16.15^\circ$	6. $\theta \approx 37.26^\circ$
7. $\theta \approx 32.39^\circ$	8. $x \approx 9.08$
9. $\theta = 90^\circ$	10. $x \approx 10.54$
11. 24.49	12. 22.6
13. 4.0	14. 83.3°
15. 17.15	16. 11.3
17. 23.32	18. 16.46
19. 11.04	20. 45.14
21. 9.34	22. 5.32 or 1.32
23. 53.13°	24. 8.92
25. 6.88	26. 61.28°
27. 39.03	28. 16
29. 71.38°	30. no triangle
31. 4.32	32. 5.07, 8.37
33. 5.66, 65.4°, 74.6°	34. 28.96°, 46.57°, 104.45°
35. ≈ 11.08 feet	36. ≈ 156,235,361 miles

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